

Minimal noncanonical cosmologies

Gabriela Barenboim

*Departament de Física Teòrica, Universitat de València
Carrer Dr. Moliner 50, E-46100 Burjassot (València), Spain
Email: gabriela.barenboim@uv.es*

Joseph D. Lykken

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510, U.S.A.
Email: lykken@fnal.gov*

ABSTRACT: We demonstrate how much it is possible to deviate from the standard cosmological paradigm of inflation-assisted Λ CDM, keeping within current observational constraints, and *without adding to or modifying any theoretical assumptions*. We show that within a minimal framework there are many new possibilities, some of them wildly different from the standard picture. We present three illustrative examples of new models, described phenomenologically by a noncanonical scalar field coupled to radiation and matter. These models have interesting implications for inflation, quintessence, reheating, electroweak baryogenesis, and the relic densities of WIMPs and other exotics.

KEYWORDS: Cosmology of Theories beyond the SM, Beyond Standard Model.

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1. Introduction

The recent release of the WMAP three year data [1] illustrates the extent to which cosmological model-building is now constrained and guided by precision data. It also emphasizes the extent to which a Standard Model of cosmology has emerged. This concordance model is denoted the “power-law Λ CDM model” in [1]; a slightly more theory-loaded designation is the “inflation-assisted Λ CDM model”. In this model the universe on large scales is homogeneous, isotropic, and spatially flat. The universe contains matter (dominated by dark matter), radiation, and dark energy. The dark energy consists either of a cosmological constant or of a quintessence field whose equation of state parameter $w(t)$ is close to -1 now. Primordial density perturbations have a nearly scale invariant spectrum. At scales comparable to the current Hubble radius, the scalar perturbations have a slightly negative spectral tilt [1].

A weakness of the cosmological data set is that direct observations only cover the era in which the scale factor $a(t)$ was about 0.001 or larger. The success of Big Bang Nucleosynthesis (BBN) suggests that the the universe was strongly radiation-dominated at $a \simeq 10^{-10}$, and that not much entropy was added to the radiation bath after the time of BBN. Beyond this, and especially in order to describe earlier times, we have to add additional theoretical baggage. A primordial epoch of inflaton-driven accelerated expansion provides an attractive (though perhaps not unique) explanation for many features of cosmological data. Thus the assumption of an inflaton seems like a minimal theoretical input allowing us to make models of early time cosmology that can be confronted with data.

Having allowed for the possibility of an inflaton and a quintessence field, it is important to ask: what is the general class of such models currently allowed by data? The purpose of this paper is to begin to answer this question. We will do this in a purely phenomenological framework. Thus we avoid the impossible task of trying to classify all possible additional top-down theoretical assumptions, and are able to discuss a wide variety of models in a single framework.

Our aim is to demonstrate how much it is possible to deviate from the standard cosmological paradigm, keeping within current observational constraints, and *without adding to or modifying any theoretical assumptions*. We will show that within a simple framework there are many new possibilities, some of them wildly different from the standard picture. Since these possibilities do not involve any new theoretical assumptions or inputs, Occam's razor does not separate them from the standard paradigm.

To be completely concrete, let us restate the theoretical assumptions that underlie standard cosmology:

1. The large scale evolution of the universe is well-described by solutions of the four dimensional Einstein equations which are spatially homogeneous and isotropic (FRW). This assumption holds for any prior epoch when the temperature is (very roughly) less than M_{Planck} . During this time the universe has been expanding monotonically.
2. The primordial distribution of light nuclei was produced by the standard BBN processes. There is, of course, strong observational evidence supporting this assumption.
3. The solution to the horizon and flatness problems is provided by inflationary expansion in earlier epochs. This process is parametrized by a single scalar inflaton, which also produces primordial scalar and curvature perturbations which are nearly scale invariant, consistent e.g., with observations of the Cosmic Microwave Background (CMB).
4. The dominant form of matter now is cold dark matter, consistent with a total Ω indistinguishable from unity, and thus with the above assumption of primordial inflation.
5. Large scale evolution has lately undergone a transformation from a matter-dominated expansion to an accelerating expansion. The source of this accelerating expansion is parametrized either by a small positive cosmological constant, or by a quintessence scalar whose equation of state parameter $w(t)$ is close to -1 now.

We regard the above assumptions as minimal, and will refer to them in this paper as *the minimal standard assumptions*. However we wish to attack the usual conclusion, i.e. that the minimal standard assumptions imply a cosmological history of the universe essentially identical to the standard picture of inflation-assisted Λ CDM cosmology. Our purpose is to demonstrate that current data allow a much richer range of possibilities, even within this conservative set of assumptions. We will exhibit several examples, all within an identical framework. One of these examples is superficially similar to the standard picture,

but differs in one important feature. The other two are qualitatively very different from the standard picture and from each other. Future observations, or better analysis of existing data, may be able to distinguish between these alternative histories.

In the next section we describe a general framework for constructing models which satisfy the minimal standard assumptions. This framework parametrizes FRW cosmological histories in terms of a single scalar field θ with a potential $V(\theta)$ and (in general) a noncanonical kinetic function $F(\theta)$. In section 3 we compare this framework to a minimal approach to inflation, in which an inflaton scalar drives a primordial epoch of accelerated expansion, and generates a nearly scale invariant spectrum of scalar and curvature perturbations. As widely acknowledged, without further theoretical assumptions cosmological data do not yet distinguish whether the inflaton is a fundamental degree of freedom or merely an effective description of other physical processes. Thus we propose a much larger class of cosmological histories, that can likewise be described phenomenologically as driven by a scalar field. This larger class is constrained by data but involves no new kinds of theoretical inputs. Any particular model is fully specified by $V(\theta)$ and $F(\theta)$ combined with some prescription for how the θ field couples to ordinary radiation and matter.

In section 4 we describe three examples of new models which obey the minimal standard assumptions, described phenomenologically in the noncanonical framework. The first model resembles the standard picture, with both the inflaton and the quintessence scalar identified with θ . There is a long epoch of primordial inflation, enough to solve the horizon and flatness problems. However inflation is less rapid than in the standard picture, and the temperature decreases much more slowly, obviating the need for a period of strong reheating. The second example is the “slinky” model already presented in [2]. In this model a somewhat abbreviated epoch of primordial inflation is followed by a second period of inflation which begins just before the electroweak phase transition (EWPT), and ends before BBN. Again the radiation temperature decreases much more slowly than in the standard picture, and there is no period of large reheating. The Hubble expansion rate is much smaller during the EWPT; for Higgs sectors such that the phase transition is first order, this will enhance electroweak baryogenesis. In the third example there are many inflationary epochs; the current accelerated expansion marks the beginning of the sixth period of inflation. The fifth period of inflation occurs after BBN, but without upsetting the baryon to photon ratio. Other periods of inflation occur before and after the EWPT, also affecting the relic density of WIMPs, gravitinos, and modulinos, if present.

2. A noncanonical framework for FRW cosmology

It is not widely appreciated that any FRW cosmological history can be parametrized by a single real scalar field $\theta(x^\mu)$ coupled to Einstein gravity. In general the scalar field action will be noncanonical, meaning that it has the form

$$\int d^4x \sqrt{-g} \left[\frac{1}{2} F(\theta) P(X) - V(\theta) \right], \tag{2.1}$$

where $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta$. When $V = 0$ and $P(0) = 0$, this is the class of noncanonical theories which generate k-essence models of inflation [3]–[5]. When $P(X) = X$, as will be

true in all of our examples, (2.1) reduces to

$$\int d^4x \sqrt{-g} \left[\frac{1}{2} F(\theta) g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - V(\theta) \right]. \quad (2.2)$$

Of course one can transform such noncanonical scalars into a canonical ones via a field redefinition, but this is not useful if (as occurs in ours models) the vacuum expectation value of $F(\theta)$ vanishes at certain times.

An FRW cosmological history is completely specified by a scale factor $a(t)$, where without loss of generality we can take the scale factor now to be unity: $a_0 = 1$. Thus the cosmological history is equivalently specified by the Hubble parameter $H(t) \equiv \dot{a}/a$. Since we assumed that the FRW expansion is monotonic, we can trade the co-moving time variable t for the scale factor a . Thus an FRW cosmological history is fully specified by some $H(a)$.

Given any such history, we can immediately write down a noncanonical scalar theory which reproduces it:

$$\theta(a) = -b \ln a, \quad (2.3)$$

$$F(\theta) = \frac{3k^2}{2b^2} (1 + w(\theta)), \quad (2.4)$$

$$V(\theta) = \frac{3k^2}{4} (1 - w(\theta)) H^2, \quad (2.5)$$

where $k^2 = M_{\text{Planck}}^2/4\pi$, and b is an arbitrary real parameter introduced for convenience. The equation of state parameter $w(\theta)$ of the scalar is given by:

$$1 + w(\theta) = \frac{2b}{3} \frac{H'}{H}, \quad (2.6)$$

where the prime denotes a derivative with respect to θ , using the relation (2.3).

It is easy to show that (2.3)–(2.6) define a simultaneous solution to the Friedmann equation, the equation of motion of the scalar, and the continuity equation:

$$H^2 = \frac{2}{3k^2} \rho_\theta, \quad (2.7)$$

$$0 = \ddot{\theta} + 3H\dot{\theta} + \frac{1}{F} \left(\frac{1}{2} \dot{\theta}^2 F' + V' \right), \quad (2.8)$$

$$\dot{\rho}_\theta = -3(1 + w)H\rho_\theta, \quad (2.9)$$

where ρ_θ is the scalar energy density. Note that, for convenience, we have defined θ in (2.3) such that $\theta = 0$ now.

This framework is best understood by working out some familiar examples:

2.1 Λ CDM cosmology

It is trivial to reproduce pure Λ CDM cosmology using this framework. Our input is an FRW cosmological history defined by:

$$H^2(a) = \frac{2}{3k^2} (\rho_\Lambda^0 + \rho_m^0 a^{-3} + \rho_r^0 a^{-4}), \quad (2.10)$$

where ρ_Λ^0 , ρ_m^0 and ρ_r^0 are constants. This cosmology is reproduced by a noncanonical scalar field theory with $F(\theta)$ and $V(\theta)$ given by:

$$F(\theta) = \frac{3k^2}{2b^2} \frac{\rho_m^0 e^{3\theta/b} + \frac{4}{3}\rho_r^0 e^{4\theta/b}}{\rho_\Lambda^0 + \rho_m^0 e^{3\theta/b} + \rho_r^0 e^{4\theta/b}} , \tag{2.11}$$

$$V(\theta) = \rho_\Lambda^0 + \frac{1}{2}\rho_m^0 e^{3\theta/b} + \frac{1}{3}\rho_r^0 e^{4\theta/b} . \tag{2.12}$$

These functions have a rather peculiar form, but this is not surprising since we are mocking up an FRW expansion which in reality is driven by many different components.

2.2 ultra-slow roll inflation

Standard inflationary models address an earlier FRW epoch, in which a long inflationary phase with $H(a) \sim \text{constant}$ somehow exits gracefully into a reheated radiation dominated phase. If the scalar field interactions responsible for reheating are perturbative, they can be modeled within the FRW formalism by perturbative “friction” in the coupled equations of motion of the inflation, radiation and matter. If the scalar field interactions are non-perturbative, as in the parametric resonances responsible for preheating [6]–[10], then our simple framework is not adequate, and needs to be expanded to incorporate this additional dynamics.

During the inflationary phase, scalar and curvature perturbations are produced, with power spectra that are close to scale-invariant. Ultra-slow roll inflation [11]–[13] is a toy model for this behavior, in which the inflaton potential is a constant, and the late-time limit of the FRW evolution is a pure de Sitter phase. Ultra-slow roll inflation is specified by a single free parameter V_0 :

$$H^2(a) = \frac{2V_0}{3k^2} (1 + a^6) . \tag{2.13}$$

In our general framework this corresponds to:

$$F(\theta) = \frac{3k^2}{2b^2} e^{3\theta/b} \cosh(3\theta/b) , \tag{2.14}$$

$$V(\theta) = V_0 . \tag{2.15}$$

The “slow roll” parameters ϵ , η and ξ can be computed in the noncanonical framework using the results of [14]:

$$\epsilon = k^2 \frac{1}{F} \left(\frac{H'}{H} \right)^2 = 3 \frac{a^6}{1 + a^6} , \tag{2.16}$$

$$\eta = \frac{k^2}{F} \left[\left(\frac{H'}{H} \right)^2 + \frac{1}{2} \frac{F' H'}{F H} \right] = 3 , \tag{2.17}$$

$$\xi^2 = 3(\epsilon + \eta) - \eta^2 - \frac{1}{F H^2} \left(V'' - \frac{1}{2} \frac{F'}{F} V' \right) = 3\epsilon , \tag{2.18}$$

where in each line the second equality reproduces the standard results for ultra-slow roll inflation.

2.3 hybrid inflation

Hybrid inflation [15] is usually described by two canonical scalar fields $\phi(x)$ and $\psi(x)$ with a potential given by

$$V(\phi, \psi) = \left(M - \frac{\sqrt{\lambda}}{2} \psi^2 \right) + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g \phi^2 \psi^2 . \quad (2.19)$$

Hybrid inflation has two phases [16, 17]. In the first phase, inflation occurs while $\psi = 0$ and the ϕ inflaton rolls towards a critical value ϕ_c , given by

$$\phi_c^2 = \frac{2\sqrt{\lambda}}{g} M^2 . \quad (2.20)$$

During this inflationary phase the potential reduces to

$$V(\phi) = M^4 + \frac{1}{2} m^2 \phi^2 . \quad (2.21)$$

During the second phase of hybrid inflation, the ψ field becomes unstable and inflation ends. If $\lambda \gg g^2$, then reheating proceeds perturbatively through oscillations of the ϕ field [18]. Thus this case is amenable to description by our simple framework with a single inflaton. Here we will only explicitly reproduce the first inflationary phase.

With these caveats, the FRW cosmological history of hybrid inflation is determined once we specify the parameters ϵ and η :

$$\epsilon = \frac{r_{\pm}^2}{k^2} \phi^2 ; \quad \eta = \epsilon + r_{\pm} , \quad (2.22)$$

where there are two branches of solutions given by

$$r_{\pm} = \frac{3}{2} \left[1 \mp \sqrt{1 - \frac{2}{3} \left(\frac{m^2 k^2}{M^4} \right)} \right] . \quad (2.23)$$

In our framework this is recovered by the choices:

$$F(\theta) = \frac{r_{\pm}^2}{b^2} e^{2r_{\pm}\theta/b} , \quad (2.24)$$

$$V(\theta) = M^4 \left(1 - \frac{1}{3} \frac{r_{\pm}^2}{k^2} e^{2r_{\pm}\theta/b} \right) \exp \left(\frac{r_{\pm}^2}{k^2} e^{2r_{\pm}\theta/b} \right) . \quad (2.25)$$

The potential has a rather strange form. However, in computing the power spectra of density perturbations, we are always interested in the case

$$\frac{r_{\pm}^2}{k^2} e^{2r_{\pm}\theta/b} \ll 1 , \quad (2.26)$$

in which case both the r_+ and r_- branches reduce to

$$V(\theta) = M^4 + \frac{1}{2} m^2 e^{2r_{\pm}\theta/b} , \quad (2.27)$$

as expected from (2.21).

It is easy to verify using the first equalities of (2.16), (2.17) that (2.24), (2.25) produce the slow roll parameters of hybrid inflation as given in (2.22). In hybrid inflation one is in the slow roll regime if [17]:

$$\frac{m^2 k^2}{M^4} \ll 1. \quad (2.28)$$

We have not made this assumption; more generally, our framework is not tied to the slow roll approximation.

3. What is the inflaton?

The general framework just described ties into a key dilemma of inflationary theory. While it is certainly possible to generate a variety of phenomenologically interesting FRW cosmological histories from an inflaton scalar, this in itself gives us no idea how seriously to take this scalar as a fundamental degree of freedom. Examining the inflation potential is of no help. If the potential has a simple form, then we must certainly worry that it could be a stand-in for other physical mechanisms. If the inflaton potential is complicated, then we also suspect that it may be a stand-in for a combination of effects, as was the case in our Λ CDM example of the previous section.

Reheating is a much more promising guide, since it depends on how the inflaton couples to other degrees of freedom, and to radiation in particular. Evidence for very efficient reheating or preheating could be smoking guns for resonances or other specific dynamical features. But even with a single inflaton scalar, there is a lot of freedom to mock up a variety of mechanisms which transfer energy back and forth between the scalar and radiation or matter. So we are still a long way from being able to use this information (ultimately derived from data) to reveal the true degrees of freedom behind the inflaton.

The inevitable conclusion is that presently we have little guidance for how to distinguish which inflaton potentials are more “plausible” on theoretical grounds. Instead the current thrust of inflationary theory is properly towards the bottom-up approach of reconstructing purely phenomenological inflaton potentials from data [19]–[26]. The framework that we are promoting here is a useful middle ground for linking this bottom-up approach to top-down models — models that begin with more narrow assumptions about new fundamental degrees of freedom.

From the discussion of the previous section, it is clear that many cosmologies that satisfy the minimal standard assumptions can be represented by a single noncanonical scalar coupled to radiation and matter. This includes standard inflation models that match onto Λ CDM, as well as some models of quintessential inflation. A model is specified by the functions $F(\theta)$, $V(\theta)$, and by some phenomenological friction terms in the equations of motion, which represent the interactions of this scalar with radiation and matter. Current data, and the minimal standard assumptions themselves, place rather strong constraints on which models of this type are viable.

In this general framework, as we have already seen, there is little motivation to regard some models as simpler or more plausible than others. All models that satisfy the constraints from data are equally good, in the sense that they have the same number of degrees

of freedom and satisfy the same set of theoretical assumptions. We can only distinguish among models by adding more theoretical assumptions, or adding further constraints from data.

Models of this general class are generically models of quintessential inflation; this is to say that w of the noncanonical scalar is close to -1 today, and \dot{w} is nonzero over most or all of FRW cosmological history. This kind of scenario was the motivation for the “slinky” model of quintessential inflation proposed in [2], as well as the “undulant” cosmologies discussed in [27, 28]. In fact the entire class of models that we are describing can be considered as a generalization of the particular slinky model presented in [2]. This model gives a very nonstandard cosmological history. This is our first clue that there is a rich class of new models that follow from the minimal standard assumptions.

In the next section we present three examples of new models. All of them obey the minimal standard assumptions. The first gives an FRW cosmological history which looks fairly standard, but with a simple picture for quintessence and a gentle approach to reheating. The second and third examples are very different from the standard picture, but appear to satisfy all the obvious constraints from data.

4. New models

The models are based on the following forms for $F(\theta)$ and $V(\theta)$:

$$F(\theta) = \frac{12k^2}{b^2} \sin^2 \theta ; \tag{4.1}$$

$$V(\theta) = \rho_0 \cos^2 \theta \exp \left[\frac{3}{b} (2\theta - \sin 2\theta) \right] , \tag{4.2}$$

where ρ_0 is fixed to be the dark energy density today. These choices are motivated by asking for a simple periodic behavior in the the equation of state parameter:

$$w(a) = -\cos 2\theta(a) . \tag{4.3}$$

Of course we could attempt a simpler form for $V(\theta)$ at the expense of getting a more complicated form for $w(a)$, but this gets us into the kind of top-down model-building that we are trying (for the moment) to avoid.

Our simple framework assumes that the coupling between quintessence and radiation or matter can be described perturbatively. The simplest modification of the evolution equations consistent with energy conservation is:

$$\begin{aligned} \dot{\rho}_\theta &= -3H(1+w)\rho_\theta - k_0 m_\phi (1+w)\rho_\theta , \\ \dot{\rho}_r &= -4H\rho_r + (1-f_m)k_0 m_\phi (1+w)\rho_\theta , \\ \dot{\rho}_m &= -3H\rho_m + f_m k_0 m_\phi (1+w)\rho_\theta , \end{aligned} \tag{4.4}$$

where k_0 and f_m are small dimensionless functions of time. In our examples these will just be constants, or constants multiplied by step functions. As long as θ is not near a multiple of $\pi/2$, it is a reasonable approximation to make the replacement [2]

$$k_0 m_\phi \rightarrow kH , \tag{4.5}$$

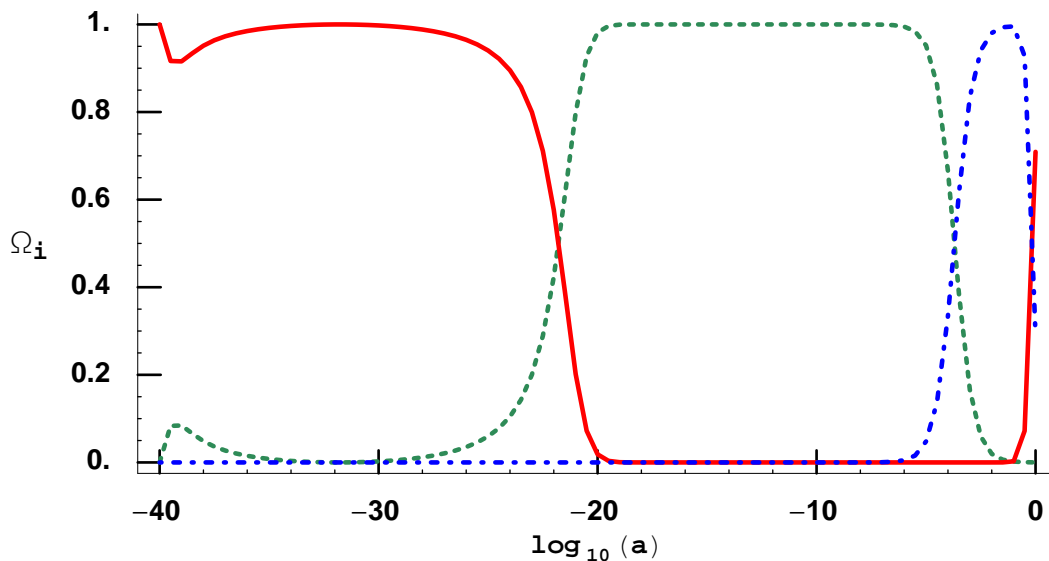


Figure 1: The cosmological history of Model 1. Shown are the relative energy density fractions in radiation (green/dashed), matter (blue/dot-dashed), and the noncanonical scalar (red/solid), as a function of the logarithm of the scale factor.

where k is another small dimensionless parameter. This replacement decouples the θ evolution equation from the Friedmann equation, giving an immediate analytic solution:

$$\rho_\theta(a) = \rho_0 \exp \left[\frac{1}{b} (3 + k) (2\theta - \sin 2\theta) \right]. \quad (4.6)$$

We have used this convenient but nonessential approximation in generating the figures shown below.

Models of this type thus have four adjustable parameters: b , k , f_m , and $a_i = a(t_i)$, with t_i the initial time at which we begin the FRW evolution. Two of these parameters, k and f_m , are essentially fixed by requiring that the matter and radiation fractions today agree with data. We do not distinguish between the excess of baryonic matter density and the much larger excess of dark matter density, thus f_m refers to the production of dark matter. Here we are assuming that the dark matter results from decay of the noncanonical scalar. If instead the dark matter is a thermal relic, then one can set $f_m = 0$ and obtain the dark matter relic density by standard methods. Note however that such calculations must take into account the nonstandard expansion histories in our models.

The remaining parameters b and a_i are constrained by several phenomenological requirements:

- Since we assume standard BBN to explain the abundances of light nuclei, the universe should be radiation dominated when the temperature is in the MeV range. In addition, we should not produce very much entropy, in the form of radiation from reheating, at any time after BBN.

- To solve the horizon problem, the ratio of the comoving horizon to the comoving Hubble radius, as measured today, should be greater than one:

$$aH \int_0^a \frac{da'}{a'} \frac{1}{a'H(a')} > 1. \quad (4.7)$$

- The scalar spectral index, for perturbations which are now on scales comparable to the Hubble radius, should be close to the WMAP-preferred value [1].
- The temperature $T = (30\rho_r/\pi^2g_*)^{1/4}$ should not exceed M_{Planck} at any time after t_i .
- Late time variations of $w(a)$ should not interfere with structure formation, or cause too much distortion of the imprint from baryon acoustic oscillations (BAO).
- The coupling of the scalar to radiation and ordinary matter must be very suppressed at late times, to satisfy bounds on Equivalence Principle violations, Faraday rotation of light from distant sources, and time variation of Standard Model parameters [29, 30].

While b and a_i are nontrivially constrained by the above considerations, many solutions remain. The tunings required are not very strong; all three of our examples were obtained from a few minutes of hand-tuning, not from a systematic parameter scan.

4.1 A simple model of quintessential inflation with gentle reheating

This model is defined by:

$$b = 1/11.7, \quad k = 0.14 \theta(10^{-10} - a), \quad f_m = 3 \times 10^{-20}, \quad a_i = 10^{-40}, \quad (4.8)$$

where the step function $\theta(10^{-10} - a)$ turns off the coupling of the scalar to radiation and matter from BBN time until now. This choice is motivated by the need to suppress the coupling of the scalar to Standard Model fields at late times, but this particular implementation is just an example from a class of similar models. The parameter f_m is very small, as is to be expected. It was already noted above that we can set $f_m = 0$ if we have in mind thermal relic dark matter of some specified variety.

This model resembles the standard paradigm in many respects. The FRW history begins with a single long period of inflation. This crosses over to a long epoch of radiation domination beginning at a temperature of about 10^9 GeV. At late times, before recombination, a matter dominated phase begins. A new accelerated expansion is beginning now. The horizon problem is solved because the ratio of the comoving horizon to the comoving Hubble radius, as measured today, is about 3.

On the other hand, the temperature history of this model is quite nonstandard. Due to the relatively large value of the parameter k , radiation and the noncanonical scalar track [31] each other. The maximum temperature is about 2×10^{15} GeV, which is rather high but still much less than M_{Planck} . As can be seen from figure 2, there is no dramatic reheating phase. Instead, the temperature falls off very slowly during the long inflationary phase, due to the tracking behavior.

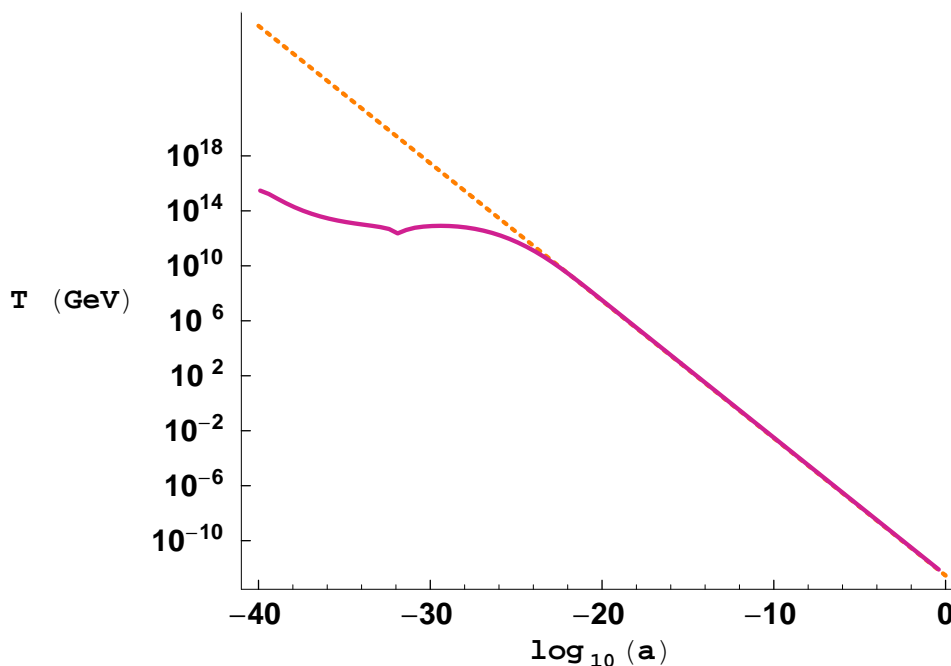


Figure 2: The temperature history of Model 1 (violet/solid), compared to a purely radiation-dominated cosmology (yellow/dashed).

It is difficult to extract a precise prediction for the spectral indices of this model (or the two following), since we are never strictly in the slow roll regime. We will be content here with a rough estimate. This is obtained starting from a canonical field redefinition:

$$\phi(x) = 2\sqrt{3}\frac{k}{b} \cos \theta . \tag{4.9}$$

In terms of the canonical scalar ϕ , the potential 4.2 can be written:

$$V(\phi) \propto \phi^2 H^2(\phi) , \tag{4.10}$$

where $H(\phi)$ is the Hubble rate we would get ignoring radiation and matter. During inflation, H is approximately constant, but this is not an especially good approximation since we are not in a slow roll regime. This is similar to the oscillatory models of quintessential inflation discussed in [32]. Taking the potential in the inflationary phase to be approximated by $V \propto \phi^2$, we can estimate the scalar spectral index n_s :

$$n_s \simeq 1 - \frac{2}{N} \tag{4.11}$$

with N the number of e-folds between the Hubble radius exit and the end of the inflationary period. For our model $N = 29$, giving $n_s = .93$, in good agreement with recent observations [1].

For the two models following, inflation takes place in installments rather than during a single primordial period. However the total number of e-folds of inflation remains approximately the same, so we expect the spectral indices to be roughly the same.

$\log_{10} a$	-40	-38	-36	-34	-32	-30
$w(a)$	0.02	-0.37	-0.70	-0.92	-1.0	-0.93
T	0	$2 \cdot 10^{14}$	$3 \cdot 10^{13}$	10^{13}	$3 \cdot 10^{12}$	$8 \cdot 10^{12}$
Ω_Λ	1	0.95	0.98	0.997	0.99997	0.998
Ω_r	0	0.05	0.02	0.003	0.00003	0.002
Ω_m	0	$4 \cdot 10^{-21}$	$7 \cdot 10^{-22}$	10^{-22}	$2 \cdot 10^{-24}$	10^{-22}
$\log_{10} a$	-28	-26	-24	-22	-20	-18
$w(a)$	-0.72	-0.39	-0.01	0.37	0.70	0.92
T	$6 \cdot 10^{12}$	$2 \cdot 10^{12}$	10^{11}	$3 \cdot 10^9$	$3 \cdot 10^7$	$3 \cdot 10^5$
Ω_Λ	0.99	0.96	0.9	0.6	0.02	$7 \cdot 10^{-6}$
Ω_r	0.01	0.04	0.1	0.4	0.98	0.999993
Ω_m	$5 \cdot 10^{-22}$	$2 \cdot 10^{-21}$	10^{-20}	$3 \cdot 10^{-19}$	$5 \cdot 10^{-17}$	$5 \cdot 10^{-15}$
$\log_{10} a$	-16	-14	-12	-10	-8	-6
$w(a)$	1.0	0.93	0.71	0.39	0.004	-0.38
T	3000	30	0.3	$3 \cdot 10^{-3}$	$3 \cdot 10^{-5}$	$3 \cdot 10^{-7}$
Ω_Λ	$3 \cdot 10^{-10}$	10^{-14}	$4 \cdot 10^{-18}$	$6 \cdot 10^{-20}$	$2 \cdot 10^{-19}$	10^{-16}
Ω_r	1.0	1.0	1.0	1.0	0.99995	0.995
Ω_m	$5 \cdot 10^{-13}$	$5 \cdot 10^{-11}$	$5 \cdot 10^{-9}$	$5 \cdot 10^{-7}$	$5 \cdot 10^{-5}$	0.005
$\log_{10} a$	-4	-2	0			
$w(a)$	-0.71	-0.92	-1.0			
T	$3 \cdot 10^{-9}$	$3 \cdot 10^{-11}$	$3 \cdot 10^{-13}$			
Ω_Λ	10^{-11}	$3 \cdot 10^{-6}$	0.71			
Ω_r	0.67	0.02	0.00006			
Ω_m	0.33	0.98	0.29			

Table 1: Model 1: the relative density fractions of dark energy, radiation, and matter, as a function of the scale factor. Also shown are the temperature T in GeV, and the equation of state parameter $w(a)$.

4.2 A model with an extra inflationary epoch

This model is defined by:

$$b = 1/7, \quad k = 0.058, \quad f_m = 10^{-11}, \quad a_i = 10^{-42}. \quad (4.12)$$

This is the slinky model of [2], with a slight decrease in the parameter k to get a better fit to the WMAP preferred value for Ω_Λ .

In this model, we are currently beginning the third epoch of accelerated expansion. A second period of accelerated expansion began just before the electroweak phase transition, and ended well before BBN. The temperature history near the EWPT is shown in figure 4. Also shown is the Hubble parameter H of Model 2 normalized to the expansion rate H_{rad} for pure radiation. H_{rad} corresponds to what is assumed in the standard paradigm. Notice that for temperatures of a few GeV the expansion rate is actually somewhat larger than normal, but at higher temperatures it is much less than normal.

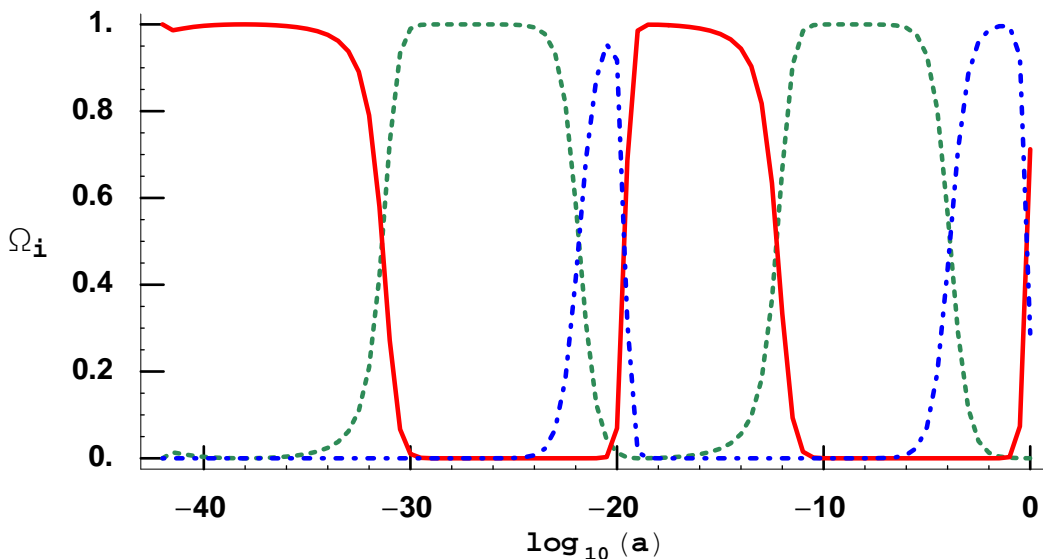


Figure 3: The cosmological history of Model 2. Shown are the relative energy density fractions in radiation (green/dashed), matter (blue/dot-dashed), and the noncanonical scalar (red/solid), as a function of the logarithm of the scale factor.

Such a nonstandard thermal history will impact on electroweak baryogenesis. For a Higgs sector such that the EWPT is first order, the change in the net baryon asymmetry is proportional to $-\log(H/H_{\text{rad}})$, where H is the expansion rate during the phase transition, and H_{rad} is the corresponding expansion rate for pure radiation [33]. If the Higgs sector is such that the EWPT is second order, the baryon asymmetry is proportional to the expansion rate [33]. Clearly one should reevaluate the popular scenarios for electroweak baryogenesis [34] in this light.

Model 2 will have major implications for predictions of the relic abundance of dark matter particles with Terascale masses. The dominant production mechanism for such particles may be scalar decays, as suggested by figure 3. Even if the dark matter particles are thermal relics, their abundance now will be affected by the nonstandard expansion rates at earlier times [35]–[38].

4.3 A model with many inflationary epochs

This model is defined by:

$$b = 0.43, \quad k = 0.33 \theta(10^{-10} - a), \quad f_m = 3 \times 10^{-14}, \quad a_i = 10^{-35}, \quad (4.13)$$

where again we have used a step function $\theta(10^{-10} - a)$ to crudely turn off the coupling of the scalar to radiation and matter at late times.

This model has many inflationary epochs, combined with strong tracking. The horizon problem is solved because the ratio of the comoving horizon to the comoving Hubble radius, as measured today, is about 3. But this is the cumulative effect of five different inflationary epochs.

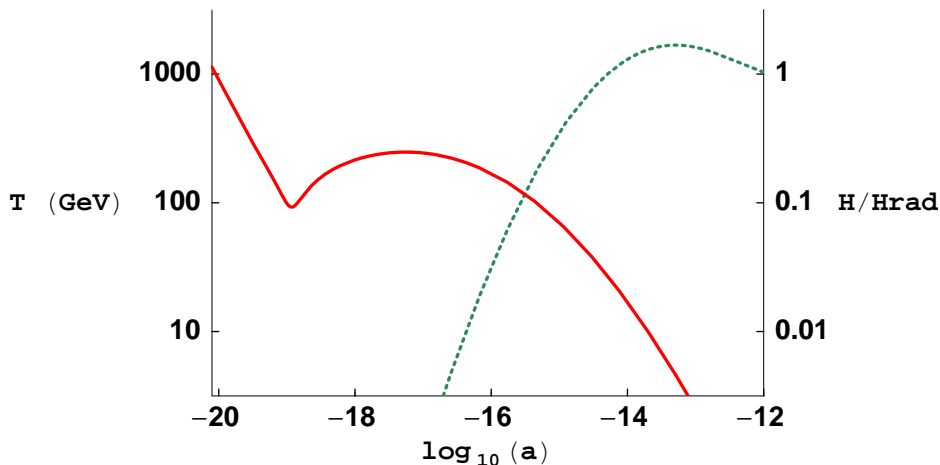


Figure 4: The temperature history of Model 2 (red/solid) near the electroweak phase transition. Shown in green/dashed is the Hubble parameter H of Model 2 normalized to the expansion rate H_{rad} for pure radiation.

$\log_{10} a$	-42	-40	-38	-36	-34	-32
$w(a)$	-0.32	-0.83	-1	-0.75	-0.19	0.45
T	0	$2 \cdot 10^{17}$	$3 \cdot 10^{16}$	10^{17}	$2 \cdot 10^{16}$	$8 \cdot 10^{14}$
Ω_Λ	1	0.997	0.99998	0.996	0.976	0.79
Ω_r	0	0.003	$2 \cdot 10^{-5}$	0.004	0.024	0.21
Ω_m	0	$5 \cdot 10^{-14}$	$3 \cdot 10^{-16}$	$5 \cdot 10^{-14}$	$4 \cdot 10^{-13}$	$2 \cdot 10^{-11}$
$\log_{10} a$	-30	-28	-26	-24	-22	-20
$w(a)$	0.90	0.98	0.64	0.04	-0.58	-0.96
T	$9 \cdot 10^{12}$	$9 \cdot 10^{10}$	$9 \cdot 10^8$	$9 \cdot 10^6$	$9 \cdot 10^4$	900
Ω_Λ	0.01	$8 \cdot 10^{-7}$	$5 \cdot 10^{-10}$	$2 \cdot 10^{-10}$	$5 \cdot 10^{-7}$	0.07
Ω_r	0.99	0.999998	0.99993	0.993	0.59	0.01
Ω_m	$7 \cdot 10^{-9}$	$7 \cdot 10^{-7}$	$7 \cdot 10^{-5}$	0.007	0.41	0.92
$\log_{10} a$	-18	-16	-14	-12	-10	-8
$w(a)$	-0.94	-0.52	0.11	0.69	0.99	0.87
T	220	170	17	0.3	$3 \cdot 10^{-3}$	$3 \cdot 10^{-5}$
Ω_Λ	0.9992	0.99	0.94	0.33	0.0002	$2 \cdot 10^{-8}$
Ω_r	0.0008	0.01	0.06	0.67	0.9998	0.9999
Ω_m	$2 \cdot 10^{-5}$	$5 \cdot 10^{-10}$	$3 \cdot 10^{-11}$	$5 \cdot 10^{-9}$	$8 \cdot 10^{-7}$	10^{-4}
$\log_{10} a$	-6	-4	-2	0		
$w(a)$	0.39	-0.25	-0.79	-1.0		
T	$3 \cdot 10^{-7}$	$3 \cdot 10^{-9}$	$3 \cdot 10^{-11}$	$3 \cdot 10^{-13}$		
Ω_Λ	10^{-10}	$2 \cdot 10^{-9}$	$7 \cdot 10^{-6}$	0.71		
Ω_r	0.992	0.56	0.01	0.00005		
Ω_m	0.008	0.44	0.99	0.29		

Table 2: Model 2: the relative density fractions of dark energy, radiation, and matter, as a function of the scale factor. Also shown are the temperature T in GeV, and the equation of state parameter $w(a)$.

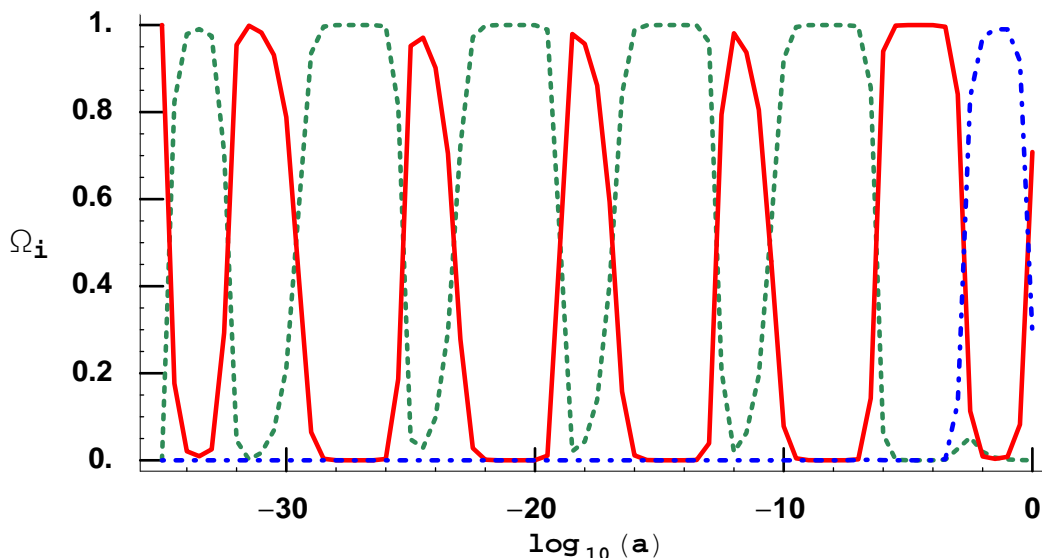


Figure 5: The cosmological history of Model 3. Shown are the relative energy density fractions in radiation (green/dashed), matter (blue/dot-dashed), and the noncanonical scalar (red/solid), as a function of the logarithm of the scale factor.

Model 3 has inflationary epochs somewhat before and somewhat after the electroweak phase transition. There is also a period of accelerated expansion which occurs after BBN and ends around the time of recombination. Certainly these effects could be important for predicting the abundances of WIMPs, gravitinos, modulinos and other exotic relics.

From table 3, one notes that that radiation fraction of Model 3 at the time of BBN is only about 0.92. This saturates the lower bound [39] required for successful BBN.

5. Outlook

We have described a simple framework for a large class of models in which a single non-canonical scalar drives quintessential inflation. Such models are minimal in the sense that they carry no additional theoretical baggage beyond the standard assumptions that we have reviewed. We have presented three examples of new models in this class. To generate the examples, we made convenient choices of $F(\theta)$, $V(\theta)$, and $P(X)$, and we took the couplings of the scalar to radiation and matter to be perturbative, which simplifies the analysis. This framework could be made more robust by, e.g., including the possibility of preheating, or looking at examples with nontrivial $P(X)$. By modifying the arbitrary forms used in (4.1) and (4.2), it should be possible to connect our framework to a number of existing top-down models. In this context it would also be interesting to examine the intrinsically noncanonical features, such as the strong coupling behavior associated with $F(\theta) \rightarrow 0$.

Our three examples look very different from standard inflation-assisted Λ CDM cosmology. We would expect therefore that observational data can discriminate among them. It may be that a more detailed comparison with existing data is sufficient to rule out all

$\log_{10} a$	-34	-32	-30	-28	-26	-24
$w(a)$	0.63	-0.96	0.14	0.85	-0.82	-0.20
T	$4 \cdot 10^{16}$	$4 \cdot 10^{14}$	$2 \cdot 10^{14}$	$2 \cdot 10^{12}$	$2 \cdot 10^{10}$	10^9
Ω_Λ	0.02	0.95	0.79	0.0002	0.003	0.9
Ω_r	0.98	0.05	0.21	0.9998	0.997	0.1
Ω_m	$2 \cdot 10^{-13}$	$8 \cdot 10^{-13}$	10^{-14}	$3 \cdot 10^{-12}$	$3 \cdot 10^{-10}$	$2 \cdot 10^{-12}$
$\log_{10} a$	-22	-20	-18	-16	-14	-12
$w(a)$	0.98	-0.58	-0.52	0.99	-0.27	-0.77
T	$2 \cdot 10^7$	$2 \cdot 10^5$	8000	300	3	0.04
Ω_Λ	0.001	0.0003	0.96	0.01	0.00005	0.98
Ω_r	0.999	0.9997	0.04	0.99	0.99995	0.02
Ω_m	$2 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$6 \cdot 10^{-10}$	$9 \cdot 10^{-9}$	$9 \cdot 10^{-7}$	$3 \cdot 10^{-7}$
$\log_{10} a$	-10	-8	-6	-4	-2	0
$w(a)$	0.89	0.07	-0.94	0.68	0.40	-1.0
T	$3 \cdot 10^{-3}$	$3 \cdot 10^{-5}$	$3 \cdot 10^{-7}$	$3 \cdot 10^{-9}$	$3 \cdot 10^{-11}$	$3 \cdot 10^{-13}$
Ω_Λ	0.08	0.00002	0.94	0.9998	0.01	0.71
Ω_r	0.92	0.99993	0.06	0.0001	0.02	0.00006
Ω_m	$5 \cdot 10^{-7}$	0.00005	0.0003	0.0001	0.97	0.29

Table 3: Model 3: the relative density fractions of dark energy, radiation, and matter, as a function of the scale factor. Also shown are the temperature T in GeV, and the equation of state parameter $w(a)$.

three models. However we would then expect that minor modifications of these models would make at least some of them viable again.

All of this emphasizes the importance of understanding TeV cosmology from independent physics inputs, as has been done so successfully with MeV cosmology and Big Bang Nucleosynthesis. A common feature of our nonstandard cosmologies is that they affect both the electroweak phase transition (at $T \sim 100$ GeV) and the abundance of weakly interacting dark matter components. Most theory papers on the EWPT or dark matter abundances simply assume standard cosmology, which is certainly dangerous. A better strategy is to regard TeV cosmology as one of the important *outputs* of particle physics. This will require digesting the results of the next generation of experiments at colliders, direct dark matter searches and other experiments and observations.

The goal should be a “TeV signpost” as constraining to cosmological model building as BBN is now. This may be the only robust way to rule out (or rule in) the type of nonstandard scenarios that we have presented here.

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